Contents at left – click to preview chapters.

If contents are not visible use your toolbar above and click on the indicated button (to open table of contents):

STANDARDS BASED MCAS

Secondary Mathematics (Middle School)

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- Review materials organized by curriculum standard.
- Instruction, not fill in the blanks.
- Basic skills practice.
- Internet resources aligned to curriculum standards.

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Next Page - use page number arrows below to navigate.

The Strands and Standards in Visual Outline
There are Five Math Strands

Strand One: Number Sense and Operations – Exploratory Concepts

- Understand Numbers, ways of representing numbers, relationships among numbers, and number systems.
- Understand meanings of operations and how they relate to one another.
- Compute fluently and make reasonable estimates.

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Standard One: Compare, order, estimate, and translate among integers, fractions and mixed numbers (i.e., rational numbers), decimals, and percents.

↓

Standard Two: Define, compare, order, and apply frequently used irrational numbers, such as $\sqrt{2}$ and π.

↓

Standard Three: Use ratios and proportions in the solution of problems, in particular, problems involving unit rates, scale factors, and rate of change.

↓

Standard Four: Represent numbers in scientific notation, and use them in calculations and problems involving unit rates, scale factors, and rate of change.

↓

Standard Five: Apply number theory concepts, including prime factorization and prime numbers, to the solution of problems.

↓

Standard Six: Demonstrate an understanding of absolute value, e.g., $|-3| = |3| = 3$.

↓

Standard Seven: Apply the rules of powers and roots to the solution of problems. Extend the Order of Operations to include positive integer exponents and square roots.

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Standard Eight: Demonstrate an understanding of the properties of arithmetic operations on rational numbers. Use the associative, commutative, and distributive properties; properties of the identity and inverse elements (e.g., $-7 + 7 = 0$; $\frac{3}{4} \times 4/3 = 1$); and the notion of closure of a subset of the rational numbers under an operation (e.g., the set of odd integers is closed under multiplication but not under addition).

Standard Nine: Use the inverse relationships of addition and subtraction, multiplication and division, and squaring and finding square roots to simplify computations and solve problems, e.g., multiplying by ½ or 0.5 is the same as dividing by 2.

↓

Standard Ten: Estimate and compute with fractions (including simplification of fractions), integers, decimals, and percents (including those greater than 100 and less than 1).

↓

Standard Eleven: Determine when an estimate rather than an exact answer is appropriate and apply in problem situations.

↓

Standard Twelve: Select and use appropriate operations – addition, subtraction, multiplication, division, and positive integer exponents – to solve problems with rational numbers (including negatives).

Strand Two: Patterns, Relations and Algebra

- Understand patterns, relations and functions.
- Represent and analyze mathematical situations and structures using algebraic symbols.
- Use mathematical models to represent and understand quantitative relationships.
- Analyze change in various contexts.

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Standard One: Extend, represent, analyze, and generalize a variety of patterns with tables, graphs, words, and when possible, symbolic expressions. Include arithmetic and geometric progressions, e.g., compounding.

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Standard Two: Evaluate simple algebraic expressions for given variable values, e.g., $3a^2 - b$ for $a = 3$ and $b = 7$.

↓

Standard Three: Demonstrate an understanding of the identity $(-x)(-y) = xy$. Use this identity to simplify algebraic expressions, e.g., $(-2)(-x+2) = 2x - 4$.

↓

Standard Four: Create and use symbolic expressions and relate them to verbal, tabular, and graphical representations.

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Standard Five: Identify the slope of a line as a measure of its steepness and as a constant rate of change from its table of values, equation, or graph. Apply the concept of slope to the solution of problems.

Standard Six: Identify the roles of variables within an equation, e.g., y = mx + b, expressing y as a function of x with parameters ma and b.

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Standard Seven: Set up and solve linear equations and inequalities with one or two variables, using algebraic methods, models, and/or graphs.

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Standard Eight: Explain and analyze – both quantitatively and qualitatively, using pictures, graphs, charts, or equations – how a change in one variable results in a change in another variable in functional relationships, e.g., $C = \pi d$, $A = \pi r^2$ (A as a function of r), $A_{rectangle} = lw$ ($A_{rectangle}$ as a function 1 and w)

↓

Standard Nine: Use linear equations to model and analyze problems involving proportional relationships. Use technology as appropriate.

↓

Standard Ten: Use tables and graphs to represent and compare linear growth patterns. In particular, compare rates of change and x- and y- intercepts of different linear patterns.

Strand Three: Geometry

- Analyze characteristics and properties of two and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships.
- Specify locations and describe spatial relationships using coordinate geometry and other representational systems.
- Apply transformations and use symmetry to analyze mathematical situations.
- Use visualization, spatial reasoning, and geometric modeling to solve problems.

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Standard One: Analyze, apply, and explain the relationships between the number of sides and the sums of the interior and exterior angle measures of polygons.

↓

Standard Two: Classify figures in terms of congruence and similarity, and apply these relationships to the solution of problems.

↓

Standard Three: Demonstrate an understanding of the relationships of angles formed by intersecting lines, including parallel lines cut by a transversal.

↓

Standard Four: Demonstrate an understanding of the Pythagorean Theorum. Apply the theorum to the solution of problems.

Standard Five: Use a straight-edge, compass, or other tools to formulate and test conjectures, and to draw geometric figures.

↓

Standard Six: Predict the results of transformations on unmarked or coordinate planes and draw the transformed figure, e.g., predict how tessellations transform under translations, reflections and rotations.

↓

Standard Seven: Identify three-dimensional figures (e.g. prisms, pyramids) by their physical appearance, distinguishing attributes, and spatial relationships such as parallel faces.

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Standard Eight: Recognize and draw two-dimensional representations of three-dimensional objects, e.g., nets, projections, and perspective drawings.

Strand Four: Measurement:

- Understand measurable attributes of objects and the units, systems, and processes of measurement
- Apply appropriate techniques, tools, and formulas to determine measurements.

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Standard One: Select, convert (within the same system of measurement), and use appropriate units of measurement or scale.

↓

Standard Two: Given the formulas, convert from one system of measurement to another. Use technology as appropriate.

↓

Standard Three: Demonstrate an understanding of the concepts and apply formulas and procedures for determining measures, including those of area and perimeter/circumference of parallelograms, trapezoids, and circles. Given the formulas, determine the surface area and volume of rectangular prisms, cylinders, and spheres. Use technology as appropriate.

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Standard Four: Use ratio and proportion (including scale factors) in the solution of problems, including problems involving similar plane figures and indirect measurement.

↓

Standard Five: Use models, graphs, and formulas to solve simple problems involving rates, e.g., velocity and density.

Strand Five: Data Analysis, Statistics, and Probability

- Formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them.
- Select and use appropriate statistical methods to analyze data.
- Develop and evaluate inferences and predictions that are based on data.
- Understand and apply basic concepts of probability.

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Standard One: Describe the characteristics and limitations of a data sample. Identify different ways of selecting a sample, e.g., convenience sampling, responses to a survey, random sampling.

↓

Standard Two: Select, create, interpret, and utilize various tabular and graphical representations of data, e.g., circle graphs, Venn diagrams, scatterplots, stem-and-leaf plots, box-and-whisker-plots, histograms, tables, and charts. Differentiate between continuous and discrete data and ways to represent them.

↓

Standard Three: Find, describe, and interpret appropriate measures of central tendency (mean, median, and mode) and spread (range) that represent a set of data. Use these notions to compare different sets of data.

↓

Standard Four: Use tree diagrams, tables, organized lists, basic combinatorics ("fundamental counting principle"), and area models to compute probabilities for simple compound events, e.g., multiple coin tosses or rolls of dice.

Strand One: Number Sense and Operations

Standards 8-12: Properties of Operations

In this standard students engage in problem solving, communicating, reasoning, connecting and representing as they:

- Understand properties of operations on rational numbers.
- Use the inverse relationships of addition and subtraction, multiplication and division and squaring.
- Estimate and compute with fractions, know when an estimate is appropriate.
- Demonstrate how basic operations are related to one another.
- Select and use appropriate operations to solve problems with rational numbers (including negatives).

Prime numbers and factors:

Numbers can be said to be prime or composite. A prime number is a whole number greater than 1 with exactly two factors, 1 and the number itself. It is divisible by no other number but itself. Primes include:

2, 3, 5, 7, 11, 13...

A composite number is a whole number greater than 1 with more than two factors, 1, itself and another number: Examples of composite numbers:

4, 6, 8, 9, 10, 12, ...

Exercise:

- If you divide the temperature today by 7, the remainder is 1.
- If you divide the temperature by 2, the remainder is 1.
- If you divide the temperature by 3, the remainder is 1.
- The temperature is not divisible by 5.
- Now you know how cold or hot it is. What's today's temperature?

If the temperature is divisible by 7, 2, and 3, it can be found by multiplying its factors: 7 x 2 x 3 = 42.

To get a remainder of 1 on each division, the temperature must be 42 + 1 or 43.

No other temperature fits this description.

Another exercise:

I'm making Christmas wreaths. I have 36 miniature pinecones, 48 tiny silver ornaments and 60 small silk poinsettias. I want to make the wreaths so that they all have the same number of pinecones, ornaments and poinsettias. What is the greatest number of wreaths I can make?

To find out, factor each number.

- pinecones: 36: (1, 2, 3, 4, 6, 9, 12, 18, 36)
  - these are all the numbers by which 36 can be divided.
- ornaments: 48: (1, 2, 3, 4, 6, 8, 12, 16, 24, 48)
- poinsettias: 60: (1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60)

Find the common factors:

1, 2, 3, 4, 6, 12

The Greatest Common Factor is 12 (GCF). I can make twelve wreaths. This technique can be used to solve some word problems that are found on the MCAS test.

Try some exercises to cement this idea:

1. 18 is not a factor of 84 because:

   A. 84 is not a prime number.
   B. 18 is not a prime number.
   C. 6 is not a factor of 84.
   D. 9 is not a factor of 84.

Hint: Factors of 84 are: 1, 2, 3, 4, 6, 7, 12, 14, 42

If you are still unclear about factorization, consult your math textbook or visit the Internet at:

http://mathforum.org/dr.math/dr-math.html

Also, consider these rules for factoring the number 642:

642 = 321 × 2

- two is prime circle it as a factor.

107 x 3 = 321

- 3 is prime, 107 is prime, circle them.

factors of 642 are 2, 3, 107

This process can be used for many things, especially finding least common denominator (LCD).

Strand One: Number Sense and Operations

Exercises and Word Problem Strategies

The following exercises will give help you become skillful with problems that prepare you for the MCAS Math test. In 2000 there were two problems in this area, in 2001 there were three. You will notice that many of the problems are related to fractions and the concepts of Least Common Denominator (LCD) and Least Common Multiple (LCM). You will see problems where you must translate fractions into their other forms, decimals, percents, mixed numbers. We will see some ratios. We will experience number concepts, factoring.

1. The sum of three odd numbers is:
    a. Always even.
    b. Always odd.
    c. Always a prime number.
    d. Sometimes odd.

2. In a group of 28 people, 12 are Asian. What part of the group is not Asian?
    a. 3/7
    b. 75
    c. 50
    d. 4/7

3. What is the value of p in the sequence 3, 6, 18, 72, p, 2160?
    a. 75
    b. 144
    c. 350
    d. 360

4. How many numbers between 100 and 200 are divisible by three?
    a. 32
    b. 33
    c. 34
    d. 66

5. Maggie would like to buy a book that costs $12.95 or a CD that costs $8.95 on sale. She has $10.52. Which can she afford to buy?

6. Which runner had a race time greater than 10.273?

Runner	Time
Sal	10.153
Jim	10.27
Sharif	10.2834
Engels	10.271

7. Your final grade average is 82.7689. What is your average to the nearest tenth?

8. What is the place value of the digit 8 in 184,972?

9. What specific fact is given in this problem? A football stadium has 500 rows of seats with 65 seats in each row. Each seat sells for $34.95. How much money is made if the game is a sell out?

    a. What teams are playing
    b. The number of rows of seats
    c. The number of people at the game
    d. The cost of a hot dog

10. The matrix below lists winning times in the Indy 500. Rank the racers in order from one through 5. The fastest racer ranks one.

Racer	Time
Mario Andretti	12.59
Konstantin Slavlak	12.77
Nguyen Tien Kieu	12.56
Gustav Werner	12.64
Tom Slaver	12.38

    a. Which racers had a time better than 12.6 seconds?

11. Write a whole number that is greater than 23.01 but less that 25.9?

12. Write each expression using exponents.

    a. 4 x 4 x 4 x 4 x 4
    b. 15 cubed
    c. 5 x 5 x 5 x 5 x 5 x 5
    d. 13 x 13 x 13 x 13 x 13 x 13 x 13 x 13

13. Anson earns $3.00 an hour. In four years his hourly wage will be equal to the square of his hourly wage now. What will his hourly wage be in four years?

14. Round each number to the nearest thousand.

    a. 3,840
    b. 1,468
    c. 789,092
    d. 3,456,902

15. Often it isn't necessary to use an exact number to get your point across. How else could you say the following without really losing its meaning?
    a. Nearly 76.8% of all people in the US live on or near the coasts.
    b. Babe Ruth has a career record of 1,321 home runs.
    c. The next train to Boston leaves at 10:49 A.M.
    d. The temperature at the North Pole rarely gets higher than 12.655 degrees Fahrenheit.

16. Guesstimate the right answer: A concert hall has a total of 850 seats. There were 280 people waiting in the lobby to see the concert. If the concert hall were to fill up, how many more concertgoers would have to arrive?
    a. 280
    b. 320
    c. 570
    d. 690

17. At a department store the ratio of suits to dresses sold yesterday was 7 to 4. If 44 dresses were sold what was the total number of garments sold?
    a. 11
    b. 77
    c. 121
    d. 89

18. Melanie is meeting her friend at the rock concert. Concert tickets are $60.00. Melanie bought a new outfit for the show which cost her $60.49. She bought the CD for the group who is playing. It cost $13.95. How much did she spend on her love of music?
    a. $148.95
    b. $134.44
    c. $129.35
    d. $248.00

19. Order the following fractions from least to greatest.

3/5, 1/4, 5/6, 4/7

a. 1/4, 4/7, 3/5, 5/6
b. 1/4, 3/5, 4/7, 5/6
c. 5/6, 3/5, 4/7, 1/4
d. 1/4, 3/5, 5/6, 4/7

20. Sandy works a few hours on weekends at the nursing home. One weekend she and Mrs. Maguire walked 9/10 miles to the convenience store down the street. On the return trip, they took a shortcut that was only 4/5 miles. About how far did they travel that day in whole miles?

To round fractions before estimating their sum or difference, use the following rules:

a. If the numerator is much less that the denominator, round the fraction to 0.
b. If the numerator is about half of the denominator, round the fraction to 1/2.
c. If the numerator is a little less than the denominator, round the fraction to 1.

- In this problem they traveled 9/10 of a mile one way and 4/5 of a mile the other or 8/10 of a mile.
- This equals 17/10.
- This can be expressed as a mixed number (whole number with a fraction), or a decimal.
- 1 and 7/10 miles. Or 1.7 miles or…
- 17/10 of a mile or almost 2 miles.

You'll need to use your calculator next. You should know many glossary terms by now, if not, use those flashcards you've made either at home or at school. Also, you can find some fun practice on the following web sites: There's a fun math baseball game at:

http://www.funbrain.com/math/index2.html

Make up your own quizzes here too.

You can find some computation exercises at Ask Dr. Math and some word problem exercises:

http://www.askdrmath.com/dr.math/tocs/wordproblem.middle.html

Computation and estimation skills allow you to start manipulating numbers. Now that you know what the numbers are and how they relate to each other, you need to know how to make them work for you in meaningful ways. You know about factoring, but how can you use that information to add and subtract fractions, for instance. We'll show you.

1. Find the Least Common Multiple of (5, 7, 8, 9) or (9, 18, 100, 30)? If you couldn't answer this…

In order to add or subtract fractions, they must have the same denominator:

number on the bottom in the fraction - the total number of parts

The Least Common Denominator (LCD) for a set of fractions is the least common multiple (LCM) of their denominator.

Example One: What is the least common denominator of the fractions:

1/2, 1/9, 1/6?

Solution One:

The LCD of those fractions:

Factors of the denominators: 2 x 3 x 3 = 18.

In order to add or subtract those fractions you need to make the denominators 18. How did we do that?

You had denominators:

2, 9, 6

- The factors for 2 are 2 and 1
- The factors for 9 are 1, 3, 9
- The factors for 6 are 1, 2, 3, 6

Let's go back to your definition of our concepts least common denominator (LCD or LCM) and Greatest Common Factor.

Factors of a number divide that number so that the remainder is zero (evenly). The factors of 2 are 2 and 1. The factors of 9 are 1, 3, 9 and for 6 we have 1, 2, 3, 6.

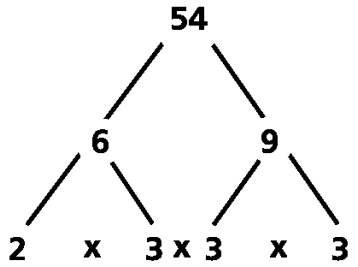
Prime factorization (the kind you need to do to find the LCM and LCD) can be described this way. Remember that a prime number is a number that can be divided by only itself and 1. A composite number has more than two factors. In our example above which of the denominators is a prime number? Only 2 because 2 has only two factors; 2 and 1.

Every composite number (the kind that has more than two factors) can be expressed as the product of prime numbers, and these are the factors you are looking for. For example:

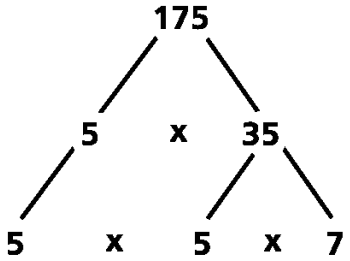
- the prime factors for 6 are 1, 2, 3.
- the prime factors for 9 are 1, 3.
- We've left out 6 and 9 in the prime factorization because they aren't prime numbers.

Let us show you a couple of factor trees that will explain this process (you can use this trick to help you - when you take the MCAS tests ask for lots of scratch paper):

The prime factorization of 54



The prime factorization of 175



We hope this tip helps you learn to find the prime factors of numbers. That skill takes you to the next, using those factors to find the LCD or LCM of denominators so you can add and subtract them.

It's important to know what the Greatest Common Factor (GCF) is too. Go to your glossary please. The Greatest Common Factor of two or more numbers is the greatest number that is a factor of both numbers.

In the case of 6 and 9 above, we had the factors of 1, 2, 3, 6 for 6 and 1, 3, 9 for 9. These are the factors, not the prime factors. The Greatest Common Factor for these two numbers is 9.

A way to do this is to find the prime factors for each number. Circle the common prime factors then find the product of those factors:

6 = 1 2 3
9 = 1 3 3
2 x 3 x 3 = 18

The Least Common Multiple of two numbers is the least nonzero number that is a multiple of each number.

- Multiples of 6 are 0, 6, 12, 18, 24, 30, 36, 42
- Multiples of 9 are 0, 9, 18, 27, 36, 45
- The Least Common Multiple of 6 and 9 = 18.

You can also use your prime factorization to find the LCM.

6 = 2 x 3 9 = 3 x 3

- Circle the common prime factors (3),
- then multiply by the remaining factors 2, 3. So 3 x 3 x 2 = 18 (which we saw above) is the least common multiple of 6 and 9.
- Let's see this more clearly:

Prime factorization:

6 = 1 x 2 x 3
9 = 1 x 3 x 3
2 x 3 x 3 = 18

Now that we have this least common multiple we can add and subtract fractions with denominators of 6 and 9

1/6 + 2/9

We're going to convert these to fractions whose denominator is 18 (LCD)

To find out what one sixth is in 18ths, we need to create equivalent fractions.

1/6 = /18

To find the new numerator to create the equivalent fraction you need to cross multiply. 6 times x = 1 x 18 or 6x = 18. Now divide 18 by 6 and you have 3. This is your new numerator.

1/6 = 3/18

Now we need to do the same thing with 2/9:

2/9 = X/18

Cross multiply 9x = 36, then divide 36 ÷ 9 = 4, so:

2/9 = 4/18

We can add now:

4/18 + 3/18 = 7/18

Congratulations, you've worked very hard. You've learned your glossary terms, you've learned about factoring, prime factorization and finding Least Common Multiple, Least Common Denominator and all those leasts and greatests. You've learned why we need to know these terms.

You can't add or subtract fractions without having a firm grip on these skills. In this standard, computation and estimation, they also want you to be able to change fractions into decimals and percents. They want you to be able to round numbers to nearest whole numbers so you can estimate answers quickly. This is an important skill in life. You probably estimate all the time in stores without even knowing it.

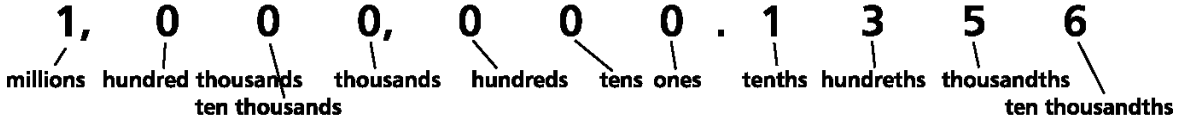
Scenario: You're in the Record Store. You see:

- a CD you want for $12.95,
- a poster for $5.00.
- You've got $15.00 in your pocket.
- Can you get both the CD and the poster?

In your head you've probably already rounded the CD price to about $13.00, then added the $5.00 for the poster. Uh oh, it comes to $18.00, not enough money.

Let's do some conversions of fractions to decimals and percents. Remember that percents are just fractions where the denominator is 100. Decimals use a dot to tell you it's time to divide a number into parts of tens.

Let's review place value a little bit. One of the charts that teachers like to put up on the wall at the beginning of each school year looks like the one below. The problem is, no one looks at it after awhile even though its message is one of the most important in math.



The dot between the ones and tenths is the decimal point and students often become intimidated when we move to the right of it. Tip: the biggest mistake made is to think that the first digit to the right of the decimal point is oneths.

When students see the number 6.75 they will often say:

"six and seventy five tenths."

There is a ones column so there should be a oneths column.

The number 6.75 is "six and seventy five hundredths". We aren't going to spend too much time on this because if you review the place value chart above many skills will come back to you.

We're going to show you some equivalent numbers.

.75	75%	75/100	75:100	75 ÷ 100.
decimal	percent	fraction	ratio	division

Again:

.63	63%	63/100	63:100	63 ÷ 100
decimal	percent	fraction	ratio	division

Let's do some work on ratios and proportions. It can be confusing. From your glossary remember: a ratio is a comparison of two similar quantities obtained by dividing one quantity by the other. Example:

$6 \div 3 = 6/3 = \underline{6:3} = 2$

The item underlined is the way we express ratios. Since a ratio is only a comparison or relation between quantities, it is an abstract number. For instance, the ratio of 6 miles to 3 miles is only 2 not two miles.

We're talking about the comparison, not the value of the division. Ratios can be written as fractions. They also have all the properties of fractions that you have learned about. The ratio of 6:3 should be stated as 2:1 as if you're reducing a fraction to its lowest terms.

Common usage has shortened the expression of ratios to be called just 2. If two quantities cannot be expressed in terms of the same unit, there cannot be a ratio between them. No comparing apples to oranges.

Time to go to the kitchen and use recipes to learn about ratios. You're using 3 cups of sugar to 5 cups of flour for those cookies? The ratio is:

3:5, 3 parts sugar to 5 parts flour.

Let's use a practical example. The football season saw the Podunk Possums win 6 games and lose 3. Their win-loss ratio was:

6:3 or 2.

Here are a few exercises so you can get some practice:
Write each ratio as a fraction in simplest terms:

A. 20 inches of snow in 15 months.
B. 36 wins to 27 losses.
C. 2 pounds of apples for $2.00.
D. 102 passengers on a sinking ship and three lifeboats.

The last item on our list for this standard is choosing the best method to solve a problem. Here are some methods for solving math problems and we guarantee you have used most of these in your math travels. We'll teach you a strategy for solving story problems.